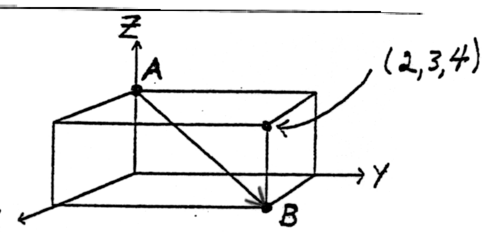
You should have a calculator. Write your name, alpha number, and section on your blue book(s) and the bubble sheet. Bubble in your alpha number in the left-most columns of the bubble sheet.

Part One. Multiple choice (50%). The first 20 problems are multiple choice. Fill in the letter of the best answer on the bubble sheet. There is no penalty for a wrong answer. YOU MUST ALSO WRITE YOUR ANSWER AND SHOW ALL YOUR WORK IN YOUR BLUE BOOK(S).

The rectangular box shown has one vertex at the origin and the opposite vertex at $(2,3,4)$. (The positive x-axis comes out of the page toward you.) The vector $\overrightarrow{AB}$ in the sketch is:

(a) $<2,3,4>$ (b) $-2\hat{\imath}+3\hat{\jmath}+4\hat{k}$ (c) $<0,2,3>$
(d) $2\hat{\imath}+3\hat{\jmath}-4\hat{k}$ (e) $<2,-3,-4>$



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2. Which of the following is a unit vector ?

(a) $<2,1,-2>$ (b) $<\frac{2}{3},\frac{1}{3},\frac{2}{3}>$ (c) $<\frac{2}{5},\frac{1}{5},\frac{2}{5}>$ (d) $<1,1,1>$ (e) $<\frac{1}{3},\frac{1}{3},\frac{1}{3}>$

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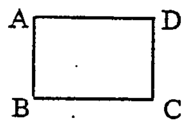
3. A, B, C and D are the vertices of the rectangle on the right.
The vector $\overrightarrow{AB}-\overrightarrow{BC}$ in the sketch equals

(a) $\overrightarrow{AC}$ (b) $\overrightarrow{DB}$ (c) $\overrightarrow{AD}$ (d) $\overrightarrow{BC}$ (e) $\overrightarrow{CB}$



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4. A vector perpendicular to both $<1,2,3>$ and $<2,1,-1>$ is

(a) $<-5,7,-3>$ (b) $<-2,1,0>$ (c) $<0,-3,2>$ (d) $<3,3,3>$ (e) $<0,1,0>$

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5. The contour map for the function $f(x,y)=\sqrt{x^2+y^2}$ is a family of

(a) paraboloids (b) cones (c) circles (d) right triangles (e) spheres

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6. An equation of the tangent plane to the surface $z=e^{2x}+\cos(y)$ at the point (0,0,2) is

(a) $z=2x+2$ (b) $x+y+z=1$ (c) $z=3x+2y$ (d) $x-y+z=0$ (e) $z=y$

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7. The maximum rate of change of the function $f(x,y)=x^2y$ at the point (3, 2) is

(a) $288/5$ (b) 1 (c) 6 (d) $12\sqrt{3}$ (e) 15

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8. When we reverse the order of integration in the iterated integral $\int_0 \int_0^{x^2} \sin(y)\,dy\,dx$, we get

(a) $\int_0^1\int_{\sqrt{y}}^1 \sin(y)\,dx\,dy$ (b) $\int_0^1\int_{\sqrt{y}}^1 \sin^{-1}(x)\,dx\,dy$ (c) $\int_0^1\int_{y^2}^0 \sin(y)\,dx\,dy$ (d) $\int_0^1\int_0^{y^2} \sin(y)\,dx\,dy$ (e) $\int_0^1\int_x^1 \cos(x)\,dx\,dy$

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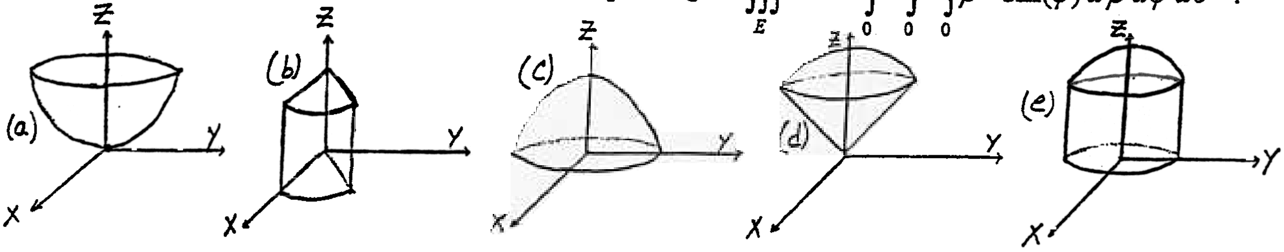
9. When we convert the iterated integral $\int_0^3\int_0^{\sqrt{9-x^2}} e^{x^2+y^2}\,dy\,dx$ from rectangular to polar coordinates, we obtain

(a) $\int_0^{\pi/2}\int_0^3 e^{r^2}\,dr\,d\theta$ (b) $\int_0^{2\pi}\int_0^3 e^{r^2}\,dr\,d\theta$ (c) $\int_0^{\pi/2}\int_0^3 re^{r^2}\,dr\,d\theta$

(d) $\int_0^3\int_0^{\sqrt{9-(r\cos(\theta))^2}} e^{r^2}\,r\,dr\,d\theta$ (e) $\int_0^3\int_0^{\sqrt{9-(r\cos(\theta))^2}} e^{r^2}\,dr\,d\theta$

10. What shape is the solid of integration E in the triple integral $\iiint_E dV = \int_0^{2\pi}\int_0^{\pi/4}\int_0^{3} \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta$?



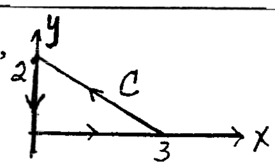
11. Suppose that $\vec{r}'(t) = <4e^{2t}, 4t, \cos(t)>$ and $\vec{r}(0) = <2,1,0>$. Then $\vec{r}(1)$ equals

(a) $<2e^2, 3, \sin(1)>$ (b) $<2e^2+2, 2, 0>$ (c) $<2e^2, 2, \sin(1)>$

(d) $<8e^2, 3, \sin(1)>$ (e) $<8e^2, 4, -\sin(1)>$

12. Use Green's Theorem to evaluate the line integral $\int_C 3y\,dx + (4x+y)\,dy$,

where C is the closed curve boundary of the triangle on the right.

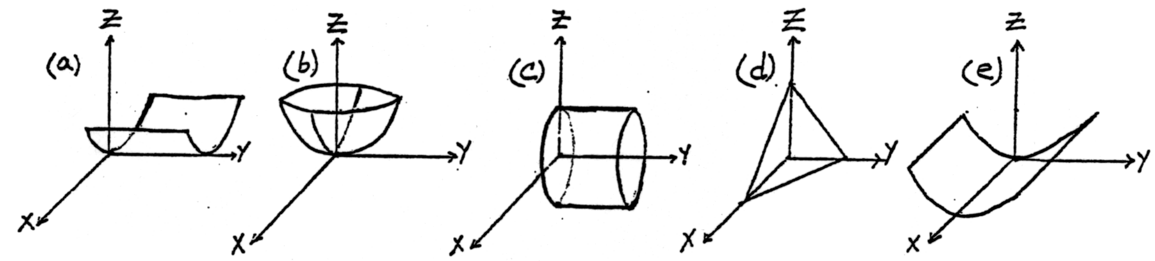


(a) 0 (b) 1 (c) 2 (d) 3 (e) 6

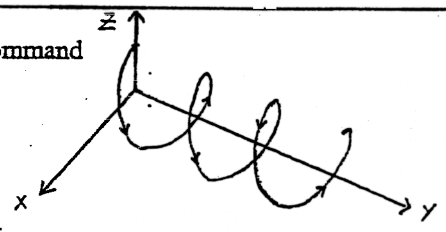
13. The line $x = 4+2t;\ y = 5-t;\ z = 2+3t:\ -\infty < t < \infty$ is perpendicular to the plane

(a) $x+5y+z=0$ (b) $2x-y+3z=5$ (c) $3x-y+2z=0$ (d) $-3x+y-2z=1$ (e) $x+y+z=0$

14. The surface in R^3 drawn by the Maple command $plot3d([x,y,y^2], x=0..1, y=-1..1);$ is



15. The curve on the right will be drawn as a result of the Maple command



(a) $plot3d([\cos(x), \sin(y), z], x = 0..6\pi, y = 0..6\pi);$

(b) $spacecurve([\cos(t), \sin(t), t], t = 0..6\pi);$

(c) $plot3d([\cos(u), \sin(u), v], u = 0..6\pi, v = 0..1);$

(d) $spacecurve([\sin(t), t, \cos(t)], t = 0..6\pi);$

(e) none of a - d

16. The function $f(s,t)$ is given by the table on the right.

$\dfrac{\partial f}{\partial s}(1,2)$ is approximately

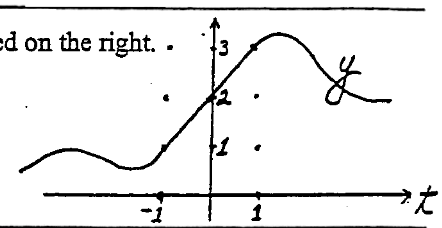
(a) -3 (b) 3 (c) -2 (d) 2 (e) 1

s \ t	0	1	2	3
0	7	4	1	-2
1	8	6	4	2
2	9	8	7	6
3	11	10	9	9

17. Let $z = x^2 + y^3$, where $x = \sin(t)$ and y is a function of t graphed on the right.

Using the chain rule to approximate $\frac{dz}{dt}$ at $t = 0$, you get

(a) 1 (b) -1 (c) 0 (d) 6 (e) 12

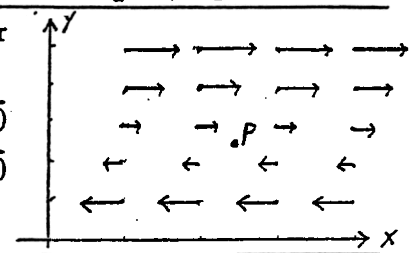


18. The vector field $\vec{F}$ is shown in the xy-plane and looks the same in all other horizontal planes. What statements are true for $\vec{F}$ at the point labeled P ?

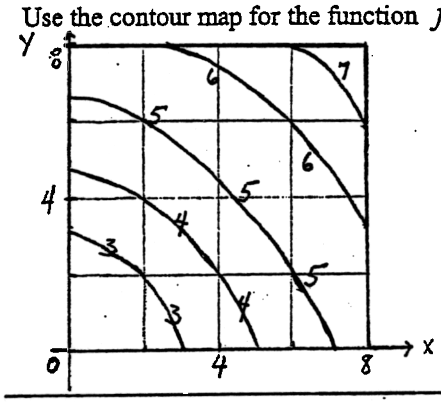
(a) $div\,\vec{F}(P) = 0,\ curl\,\vec{F}(P) \neq \vec{0}$ (b) $div\,\vec{F}(P) < 0,\ curl\,\vec{F}(P) = \vec{0}$

(c) $div\,\vec{F}(P) > 0,\ curl\,\vec{F}(P) = \vec{0}$ (d) $div\,\vec{F}(P) < 0,\ curl\,\vec{F}(P) \neq \vec{0}$

(e) $div\,\vec{F}(P) > 0,\ curl\,\vec{F}(P) \neq \vec{0}$



Use the contour map for the function $f(x,y)$ shown on the left to solve problems 19 and 20.



19. Approximate $\int_0^8 \int_0^8 f(x,y)\,dx\,dy$ by subdividing the region into four equal rectangles (i.e., $n = m = 2$) and using the midpoint rule.

(a) 16 (b) 64 (c) 100 (d) 224 (e) 304

20. Evaluate the line integral $\int_C (\vec{\nabla} f) \cdot d\vec{r}$, where C is the curve given parametrically by $x = t^2 + t,\quad y = 10 - 4t : 1 \le t \le 2$

(a) 0 (b) 1 (c) 3 (d) 4 (e) 5

Part Two. Longer Answers (50%). **SOLVE ANY 10 OF THE REMAINING 11 PROBLEMS.** They are not multiple choice. Show all of your work and put your answers in your blue book(s).

21. Consider the vectors in the plane given by $\vec{a} = \langle 4, 2 \rangle$ and $\vec{b} = \langle 1, 3 \rangle$

(a) Find the angle between $\vec{a}$ and $\vec{b}$.

(b) Find the scalar projection of $\vec{b}$ onto $\vec{a}$ (i.e. $comp_{\vec{a}}\,\vec{b}$).

(c) Find the vector projection of $\vec{b}$ onto $\vec{a}$ (i.e. $proj_{\vec{a}}\,\vec{b}$).

(d) Draw and label $\vec{a}$, $\vec{b}$ and $proj_{\vec{a}}\,\vec{b}$ on the same set of axes.

22. Consider the three points in space given by $P(1,0,1)$, $Q(2,3,4)$, and $R(2,2,2)$

(a) Find an equation for the plane going through the points P, Q, and R.

(b) Find the area of the triangle with vertices P, Q, and R.

(c) Find parametric equations for the line going through P and Q.

23. Sketch separate graphs in 3-space represented by each of the following:

(a) $x + 2y + 3z = 6$; (b) $y = x$; (c) $z = x^2 + y^2$;

(d) Sketch the solid in 3-space whose cylindrical coordinate satisfy the inequalities:

$$0 \le z \le 2,\quad \frac{\pi}{4} \le \theta \le \frac{\pi}{2},\quad 2 \le r \le 4.$$

24. Suppose that a particle has position in the plane given at time t by $\vec{r}(t) = <\cos(t),\ 2\sin(t)>$.
   (a) Find the velocity, acceleration, and speed of the particle at time $t = \pi/2$.
   (b) Sketch the path of the particle for $0 \le t \le 2\pi$ and draw the velocity and acceleration vectors for $t = \pi/2$
   (c) Find the arclength of the curve for $0 \le t \le 2\pi$ accurate to three decimal places.

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25. Consider the function $z = y - \sqrt{x}$.
   (a) Sketch the contour curves where z = 0, z = 1, and z = 2.
   (b) Find $\vec{\nabla} z(1,1)$, (gradient of z at (1,1)), and draw it on your graph from part (a).
   (c) Find the directional derivative of z at (1,1) in the direction $4\hat{i} + 3\hat{j}$.
   (d) At (1,1), what direction gives zero for the directional derivative of z ?

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26. A soccer player kicks a ball from the ground toward the center of an empty goal (no goalkeeper). The ball leaves the ground going 50 ft/sec making an angle of 60 ° with the ground. If he takes the shot from 60 feet in front of the goal and the top of the goal is 8 feet high, will the ball go in the goal ?
Start with the acceleration of the ball $\vec{a}(t)$, find its velocity $\vec{v}(t)$ and position $\vec{r}(t)$ and solve the problem.

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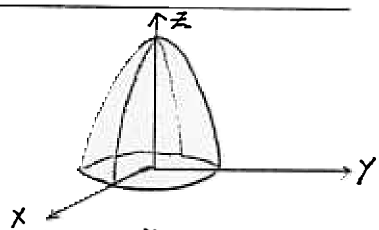
27. Consider the solid on the right bounded by the paraboloid $z = 9 - x^2 - y^2$ and the x-y plane. If it has density $\rho(x,y,z) = x^2 + y^2 + z$ (mass/vol), set up, <u>but do not evaluate</u>, a triple integral to find its mass using:
(a) rectangular coordinates, (b) cylindrical coordinates .



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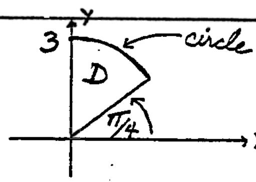
28. Set up double integrals in the requested orders over the region D on the right.
(a) $\iint f(r,\theta)\, r\, dr\, d\theta$ (b) $\iint g(x,y)\, dy\, dx$ (c) $\iint h(x,y)\, dx\, dy$



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29. (a) Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x,y,z) = <z,x,y>$ and C is the line from (1,1,1) to (3,4,5).
(b) Is the line integral in part (a) the same for any path from (1,1,1) to (3,4,5)? Explain.

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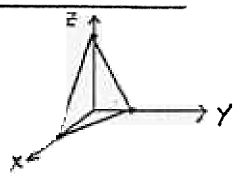
30. <u>Evaluate a surface integral</u> to find the flux, $\iint_S \vec{F} \cdot d\vec{S}$, where $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$
and S is the portion of the plane $z = 6 - 2x - 3y$ in the first octant oriented upward.



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31. (a) State the Divergence Theorem.
(b) Use the Divergence Theorem to find the total flux $\iint_T \vec{F} \cdot d\vec{S}$ for the vector field in problem 30 out of all four faces of the tetrahedron T drawn above in problem 30.
